Notes on a quantum gravitational collapse

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Everybody knows what the classical black holes are. In short, this is a spacetime region beyond the so-called event horizon. The notion of the event horizon is mathematically well defined. The situation with a definition of quantum black hole is not so clear. The problem is that the classical event horizon can be defined only globally, i.e. in order to be sure we have a black hole we would need an infinite time interval. But, in classical physics we have trajectories off all the particles and equations of motion for all the fields and can, in principle, construct some ideal models for the gravitational collapse and study the black hole formation under different conditions. In quantum theory we have no trajectory and, therefore, can not define an event horizon. Moreover, we expect (in analogy with the hydrogen atom model) the existence of bound states with discrete mass (energy) spectrum. Thus, the very process of quantum gravitational collapse should be quite different from the classical collapse.It seems reasonable to try to construct some simple quantum mechanical models in order to understand, what the definition of quantum black hole could be. Our opinion is that we need to look for some special quantum state (or states) which have such a property as the absence of "quantum hairs" (loss of the initial data memory, appearance of some entropy and all that).

In a series of papers [1] a quantum mechanical model of a self-gravitating spherically symmetric thin dust shell was constructed. Despite of simplicity, such a model reveals rather unexpected features.

First of all, it appeared that bound states (in our cases we are dealing with the s-waves only) are described by two quantum numbers (instead of one number in conventional quantum mechanics). The spherically symmetric thin dust shell motion is described by three parameters, the Schwarzschild masses m_{in} and $m_{out} = m$ inside and outside the shell, respectively, and the bare mass M. These parameters are connected to the above mentioned quantum numbers n and p by the following relations.

$$\frac{2(\Delta m)^2 - M^2}{\sqrt{M^2 - (\Delta m)^2}} = 2n\frac{m_{Pl}^2}{m}, \quad n - integer, \tag{1}$$

$$M^2 - (\Delta m)^2 = 2(1+2p)m_{Pl}^2, \quad p - integer. \tag{2}$$

Here $\Delta m = m - m_{in}$ is the total mass (energy) of the shell, and m_{Pl} is the Planckian mass. The quantum number n is reduced to the principal quantum number in the nonrelativistic Coulomb(or Kepler) problem in the corresponding limit ($\Delta m \to M - 0$). The appearance of the second quantum number p is due to the nontrivial causal structure of the Schwarzschild space-time, where we have two asymptotically flat region connected by the so-called Einstein-Rosen bridge. If the shell in its (classical) motion goes through the asymptotically flat region on "our side" of the Einstein-Rosen bridge we will call this the black hole case, if it is on

the "other side" - we will call this the wormhole case. In the first case $\frac{\Delta m}{M} > \frac{M}{4m}$ (and $\frac{\partial m}{\partial M}\mid_{n=const} > 0$ for quantum shells). For the wormhole case $\frac{\Delta m}{M} < \frac{M}{4m}$ and $\frac{\partial m}{\partial M}\mid_{n=const} < 0$. Note that from Eqs.(1) and (2) it follows that there exists the minimal possible black hole (not wormhole!) mass. It corresponds to $n = p = m_{min} = 0$ and equals $m_{min} = \sqrt{2}m_{Pl}$ for our particular model. The parameters of the shell M and Δm are imprinted in the wave functions of the bound states and can be, in principle, measured. Thus, we have "quantum hairs", and these states in no way can be considered as the quantum black hole states (zero entropy, no universality). Since the wave functions are nonzero everywhere we have a "mixture" of noncollapsing shells, collapsed shells and wormhole shells. What, then, the quantum black holes are?

Before answering this question, let us have a look at another interesting feature of our model. Due to the nontrivial causal structure of the Schwarzschild space-time we can expect the existence of one quantization condition for the unbound states as well. And this is indeed the case. For the zero bare mass $M = 0$ we have a thin null shell that can be considered as a model for radiation. And the spectrum for such a radiation is discrete and given by

$$3(\delta m)^2 + 4m\delta m = 4km_{Pl}^2, \quad k - integer, \tag{3}$$

where δm is the difference in the mass of the system before and after emission (change in Bondi's mass). The jump to the nearest level corresponds to $k = 1$. It is interesting that from the Eq.(3) it follows that the gravitating system with the mass less than $m = 2/\sqrt{5}$ cannot radiate. The spectrum for the radiation is universal in the sense that it does not depend on the structure of the radiating system. Moreover, the energies of quanta of radiation (Eq.(3)) do not match the level spacings of the shell (Eq.(3) and 3)). In order to satisfy the energy conservation law we have to suggest that together with every emitting quantum another shell(massive or massless) is created which collapses. Such a process increases the value of m_{in} inside our (initially alone) shell, thus forming the internal structure of the (future) quantum black hole and creating its entropy.

Let us now look at the shells spectrum more attentively. We see that the Eq.(2) depends only on the total mass (energy) of the shell Δm and its bare mass M, while in the Eq.(1) there enters also the inner Schwarzschild mass m_{in} ($m = m_{in} + \Delta m$). But, there exists one special level with $n = 0$ when such a dependence also disappears. If we take into account all the shell created during the quantum gravitational collapse and suppose that all of them are in the state with $n = 0$ we obtain the quantum object that "forgot" all the past like the classical black hole "forgets" all the past during classical collapse. It seems that this is just what we would like to have for the quantum black holes. Note that in such a case

$$\begin{gathered} M = \sqrt{2}m \\ m = \sqrt{2(1+2p)}m_{Pl}. \end{gathered} \tag{4}$$

To support this point of view we construct some classical model which mimics the main features of the state with $n = 0$. Let us consider the spherically symmetric distribution of perfect fluid that is initially at rest. Then, we can introduce the mass function $m(r)$ and the bare mass function $M(r)$

$$m(r) = 4\pi \int \epsilon r^2 dr, \tag{5}$$

$$M(r) = 4\pi \int \epsilon \sqrt{-g_{11}} r^2 dr, \tag{6}$$

and solve the initial value problem

$$g_{11} = -1 + \frac{2Gm(r)}{r}. \tag{7}$$

Here r is the radius (and, simultaneously, the radial coordinate), g_{11} is the radial component of the metric tensor. In order to mimic the state with $n = 0$ when nothing depends on the environment m_{in} we have to put $m(r) = ar$ and, therefore, $\epsilon = \frac{a}{4\pi r^2}$. It is interesting to note that such a distribution is marginal between the distributions $\epsilon = \frac{a}{4\pi r^{2+\gamma}}$ with $\gamma > 0$ (when near the origin there is a region where the initial state $\dot{r} = 0$ is impossible, i.e., the black hole exists from the very beginning) those with $\gamma < 0$ (when we can reach a surface with maximal area at large enough value of radius, i.e., we can construct a semi-closed world and have a wormhole case). In the case of pure dust distribution (no interaction except the gravitational one) the system would collapse. If initially the boundary of the system is r_0, then

$$\begin{aligned} m_0 &= m(r_0) = ar_0 \\ M_0 &= M(r_0) = \tfrac{ar_0}{\sqrt{1-2Ga}}. \end{aligned} \tag{8}$$

Keeping m_0 constant and increasing M_0 we get

$$\begin{aligned} &a \longrightarrow \tfrac{1}{2G} \quad \text{if} \quad M_0 \longrightarrow \infty \\ &r_0 = \tfrac{m_0}{a} \quad \longrightarrow \ 2Gm_0 = r_g. \end{aligned} \tag{9}$$

That is, limit of infinite bare mass the initial boundary lies exactly at the gravitational radius of the system. But, our quantum system is stationary. Thus, to have the static situation in our classical analogue we need to introduce an effective pressure $p(r)$. Solving the Einstein‘s equations we find that the pressure profile is the same as the energy density profile, but with different proportionality factor, namely

$$\begin{aligned} p &= \tfrac{b}{4\pi r^2}, \\ b &= \tfrac{1}{G}(1 - 3Ga - \sqrt{1-2Ga}\sqrt{1-4Ga}). \end{aligned} \tag{10}$$

We see that the coefficient b is real only $a < a_{cr} = \frac{1}{4G}$. Beyond this value b becomes a complex number. It is easy to show that at the critical value the speed of sound in our effective model reaches the speed of light. This means that even with the pressure we are unable prevent the system from collapsing! And, what is not seemed as a mere coincidence, at $a = a_{cr}$

$$M_0 = \sqrt{2} m_0 \tag{11}$$

exactly the same relation that we had before for the quantum state $n = 0$!

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